Ocean's Big Data Mining, 2014

(Data mining in large sets of complex oceanic data: new challenges and solutions)

8-9 Sep 2014 Brest (France)

Monday, September 8, 2014, 2:00 pm - 3:30 pm

Statistical Methods for detecting and attributing climate changes

Dr. Philippe Naveau, LSCE/CNRS

In this talk, our goal is to provide a review on the most used statistical methods to detect and attribute climate changes. The usual statistical framework for detection and attribution in climatology consists of a class of linear regression methods referred to as optimal fingerprinting. Three features of this regression problem are the high dimension (in space and time) with non-sparse covariance matrices, the uniqueness of the observational vector (there is only one Earth) and the limited number of numerical climate runs tainted by model error. These constrains lead to open questions concerning the choice of workable hypothesis and their associated inference schemes.

This talk would have a special emphasis on the analysis of extreme events.

About Philippe Naveau

After obtaining his PhD in Statistics at Colorado State University in 1998, Dr. Philippe Naveau was a visiting Scientist at National Center for Atmospheric Research in Boulder, Colorado for three years. Then, he was an assistant professor in the Applied Math Dept of Colorado University (2002-2004). Since 2004, he is a research scientist at the French National Research Center (CNRS) and his research work has focused on environmental statistics, especially in analyzing extremes events.

Overview of statistical methods for detecting and attributing climate changes

Alexis Hannart (CNRS) & Aurélien Ribes (Meteo-France)
& Philippe Naveau

naveau@lsce.ipsl.fr
Laboratoire des Sciences du Climat et l'Environnement (LSCE)
Gif-sur-Yvette, France

ANR-McSim, ExtremeScope, LEFE-MULTI-RISK

Statistics and Earth sciences

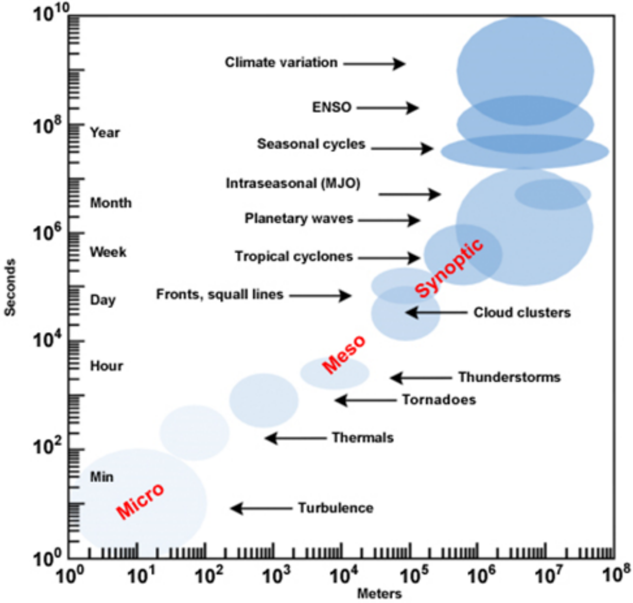
"There is, today, always a risk that specialists in two subjects, using languages full of words that are unintelligible without study, will grow up not only, without knowledge of each other's work, but also will ignore the problems which require mutual assistance".

QUIZ

- (A) Gilbert Walker
- (B) Ed Lorenz
- (C) Guillaume Maze
- (D) Rol Madden


STATISTICS
Detection &
Attribution
CLIMATE

Spatial and temporal scales in weather and climate



“Darkness” by Lord Byron

> *“The bright sun was extinguish’d and the stars did wander darkling in the eternal space, rayless, and pathless, and the icy earth swung blind and blackening in the moonless air ; Morn came and went - and came, and brought no day ...”*

Written in 1816 on the shores of Lake Geneva in the midst of the year without a summer.

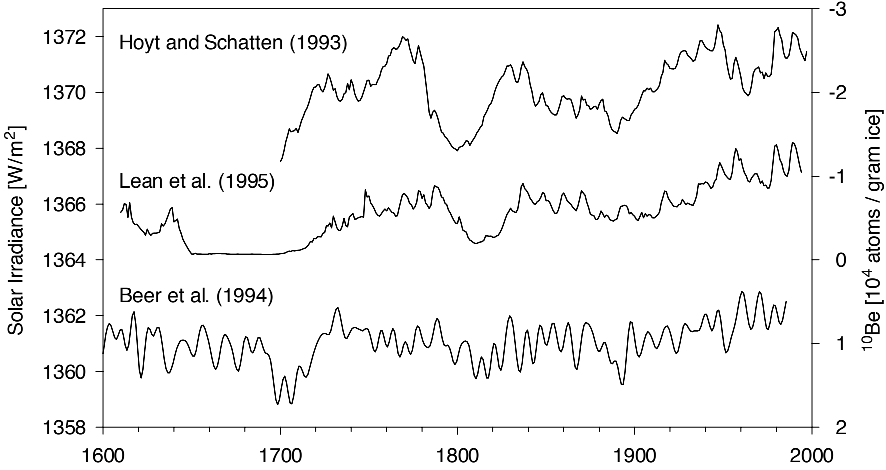
Tambora 1815 (illustrations by G. & W.R. Harlin)


- ⇒ **Plutarch** noticed that the **eruption of Etna** in 44 B.C. attenuated the sunlight and caused crops to shrivel up in ancient Rome.
- ⇒ **Benjamin Franklin** suggested that the **Laki eruption** in Iceland in 1783 was related to the abnormally cold winter of 1783-1784.

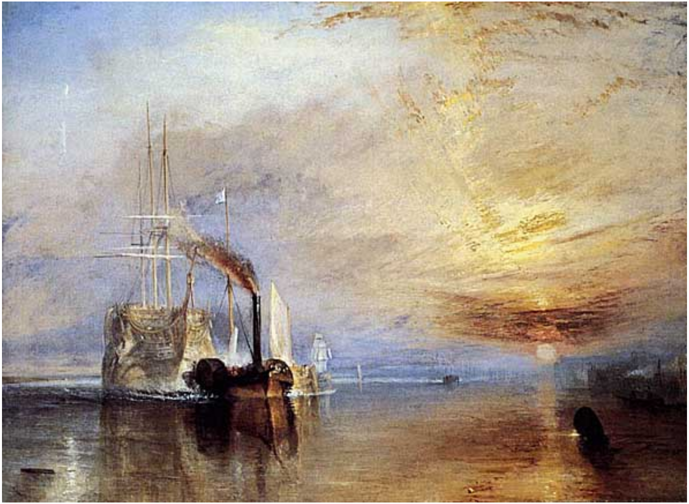
Two important natural external forcing factors :

- Solar irradiance variations (long-trend)
- Explosive volcanism : Cooling effect on climate (short-lived)

Solar forcings

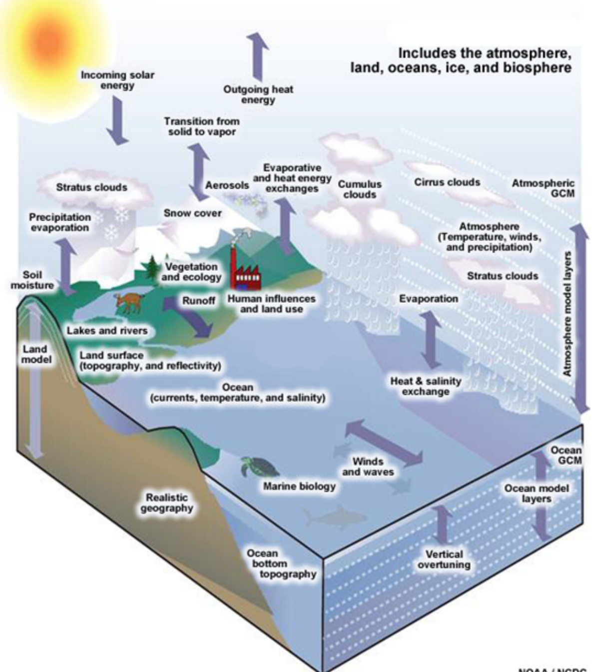


Antropogenic forcings

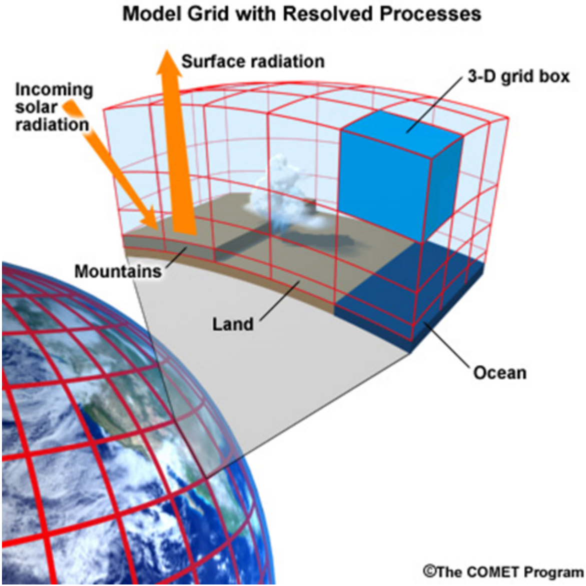


Turner, The Fighting Temeraire - tugged to her Last Berth to be broken up : 1838-39

Modeling the Climate System



NOAA / NCDC


Model Grid with Resolved Processes
Surface radiation
Incoming solar radiation
3-D grid box
Mountains
Land
Ocean
©The COMET Program

Detection

Demonstrating that climate or a system affected by climate has changed in some defined statistical sense[1] without providing a reason for that change.

IPCC Good Practice Guidance Paper on Detection and Attribution, 2010

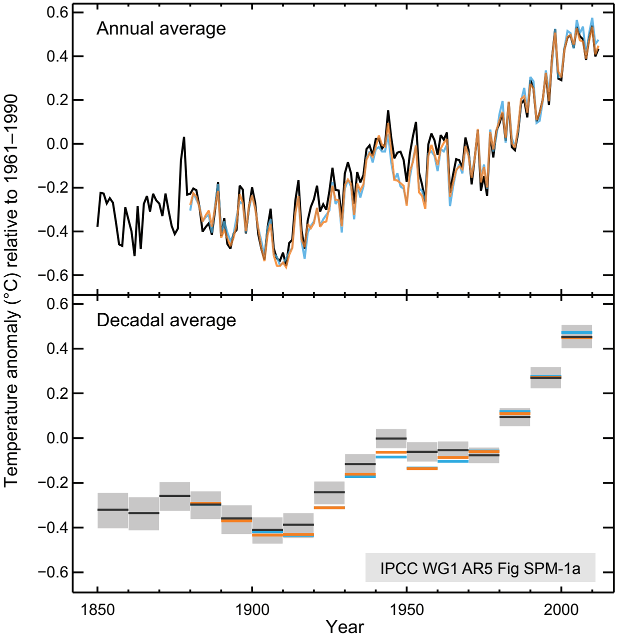
1. statistically usually, significant beyond what can be explained by internal (natural) variability alone

Examples of a "Detection" statement

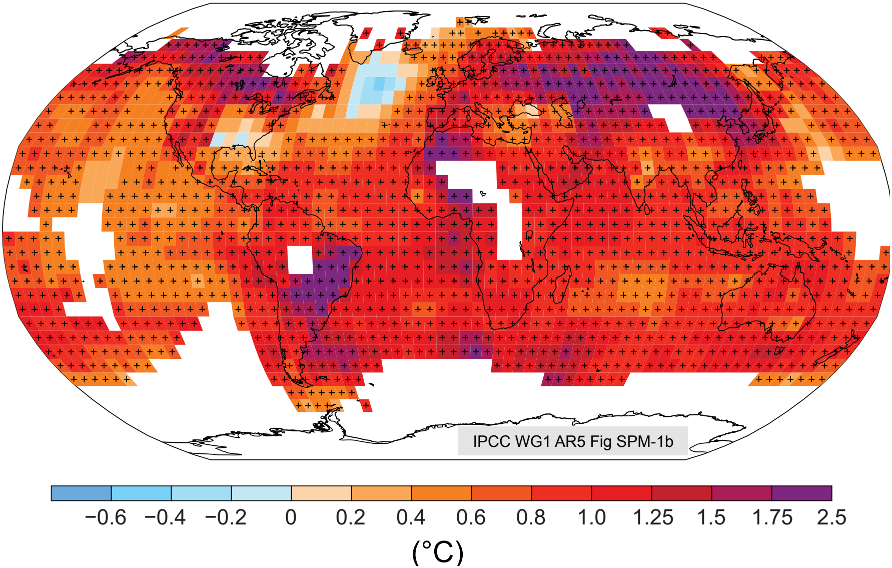
"Warming of the climate system is unequivocal, and since the 1950s, many of the observed changes are unprecedented over decades to millennia. The atmosphere and ocean have warmed, the amounts of snow and ice have diminished, sea level has risen, and the concentrations of greenhouse gases have increased."

IPCC-WG1-AR5 SPM

Observed globally averaged combined land and ocean surface temperature anomaly 1850–2012



Observed change in surface temperature 1901–2012



Examples of a “Detection” statement

These figures and statements don’t say anything about the **causes** of the observed warming.

Attribution

Evaluating the relative contributions of multiple causal factors [2] to a change or event with an assignment of statistical confidence.

2. casual factors usually refer to external influences, which may be anthropogenic (GHGs, aerosols, ozone precursors, land use) and/or natural (volcanic eruptions, solar cycle modulations

Attribution

Evaluating the relative contributions of multiple causal factors [2] to a change or event with an assignment of statistical confidence.

Consequences

Need to assess wether the observed changes are

- consistent with the expected responses to external forcings
- inconsistent with alternative explanations

2. casual factors usually refer to external influences, which may be anthropogenic (GHGs, aerosols, ozone precursors, land use) and/or natural (volcanic eruptions, solar cycle modulations

What do you need in D&A ?

Observations of climate indicators

Inhomogeneity in space and time (& reconstructions via proxies)

An estimate of external forcing

How external drivers of climate change have evolved before and during the period under investigation – e.g., GHG and solar radiation

A quantitative physically-based understanding

How external forcing might affect these climate indicators. – normally encapsulated in a physically-based model

An estimate of climate internal variability Σ

Frequently derived from a physically-based model

Classical assumptions

- Key forcings have been identified
- Signals are additive
- Noise is additive
- The large-scale patterns of response are correctly simulated by climate models
- Statistical inference schemes are efficient

Examples of a "Attribution" statement (see F. Zwiers' talk)

Attribution results

TAR (2001)

- "most of the observed warming over the last 50 years is **likely** to have been due to the increase in greenhouse gas concentrations"

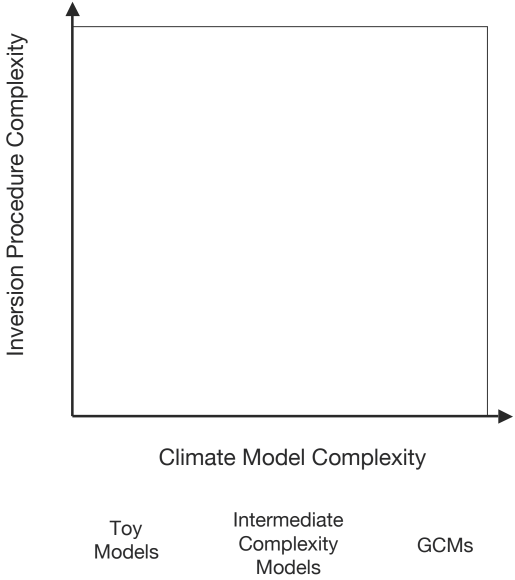
AR4 (2007)

- **likely** replaced with **very likely**
- "GHGs **likely** would have caused more warming than observed"

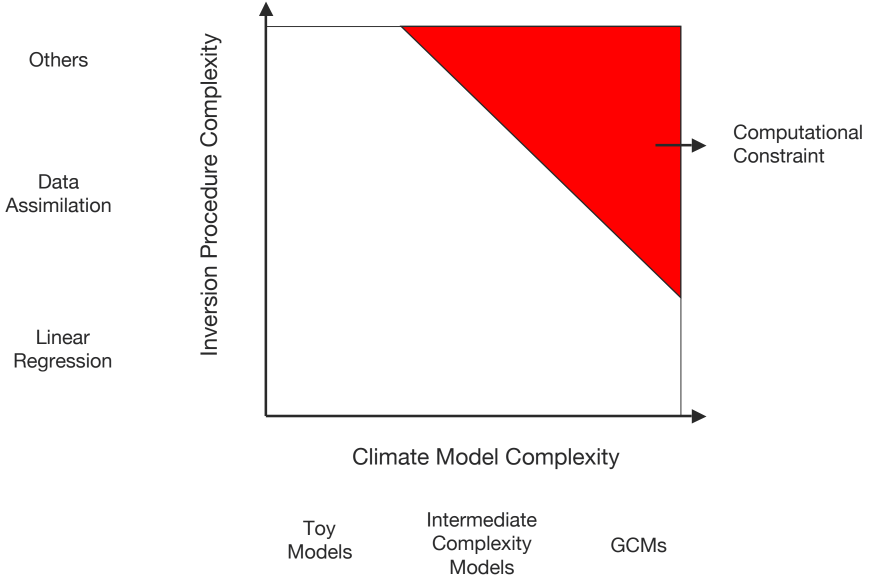
AR5 (2013)

- "It is **extremely likely** that human influence has been the dominant cause of the observed warming since the mid-20th century."
- "Greenhouse gases contributed a global mean surface warming **likely** to be in the range of 0.5°C to 1.3°C over the period 1951 to 2010 …"

Big data : statistical versus numerical models



Big data : statistical versus numerical models



Two classical statistical approaches in D&A

1- Linear regressions

- Non-optimal techniques
- Ordinary and total least square regression
- Error-in-Variables

Two classical statistical approaches in D&A

1- Linear regressions

- Non-optimal techniques
- Ordinary and total least square regression
- Error-in-Variables

2- FAR (Fraction of Attributable Risk)

The FAR = the relative ratio of two probabilities, p_0 the probability of exceeding a threshold in a "world that might have been (no antropogenic forcings)" and p_1 the probability of exceeding the same threshold in a "world that it is"

$$FAR = \frac{p_1 - p_0}{p_1}.$$

Example of an specific event, the 2003 summer heat wave over Europe.

Outline

- A quick overview
- Statistical issues
- Current solutions

One huge problem (from a stat perspective)

There is only one Earth !

One unique observation, ie. a very long vector (space $*$ time)

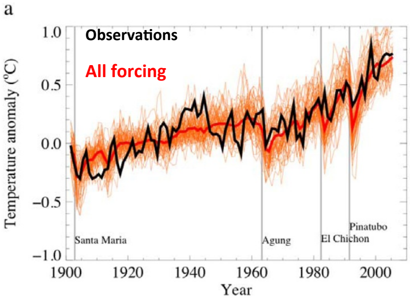
One huge problem (from a stat perspective)

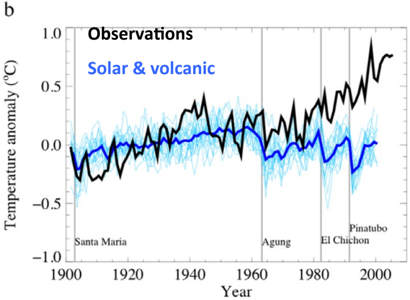
There is only one Earth !

One unique observation, ie. a very long vector (space $*$ time)

Methods based on learning from a large training set can't be easily applied

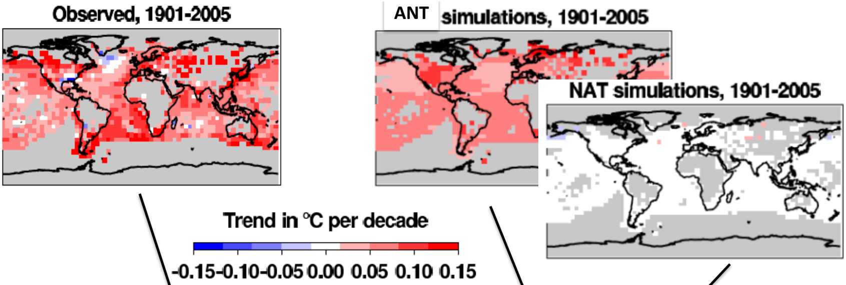
One key idea : use climate models to generate Earth's avatars





Source : Claudia Tebaldi

The basic regression scheme



$$\mathbf{Y} \qquad\qquad \mathbf{X} = (\vec{x}_{ant}, \vec{x}_{nat})$$

$$\mathbf{Y} = \mathbf{X}\beta + \varepsilon$$

Gabi Hegerl's presentation at Geneva IPCC WG1/WG2 Meeting in Sept 2009

The basic Gaussian regression scheme

$$\hat{\beta} = \left(X^T \Sigma^{-1} X\right)^{-1} X^T \Sigma^{-1} Y$$

with under the Gaussian assumption with know Σ

$$E(\hat{\beta}) = \beta \text{ and } Var(\hat{\beta}) = \left(X^T \Sigma^{-1} X\right)^{-1}$$

The basic Gaussian regression scheme

$$\hat{\beta} = \left(X^T \Sigma^{-1} X\right)^{-1} X^T \Sigma^{-1} Y$$

with under the Gaussian assumption with know Σ

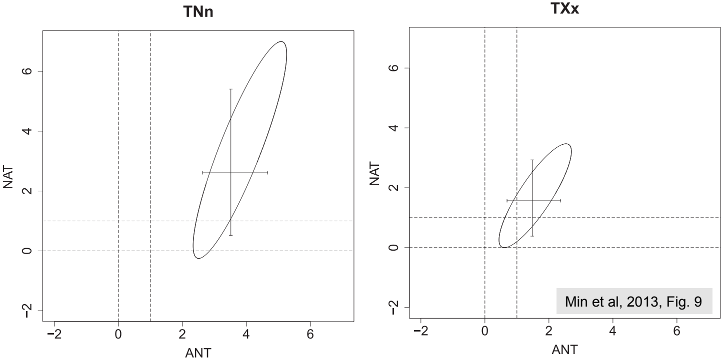
$$E(\hat{\beta}) = \beta \text{ and } Var(\hat{\beta}) = \left(X^T \Sigma^{-1} X\right)^{-1}$$

Practical questions

- $\beta = 0$ + CI ?
- $\beta = 1$ +CI ?

An example

Joint 90% confidence region for ANT and NAT detection in TNn and TXx



Details: 1951-2000 TNn and TXx from HadEX (Alexander et al, 2006), decadal time averaging, “global” spatial averaging, CMIP3 models (ANT – 8 models, 27 runs; ALL – 8 models, 26 runs; control – 10 models, 158 chunks)

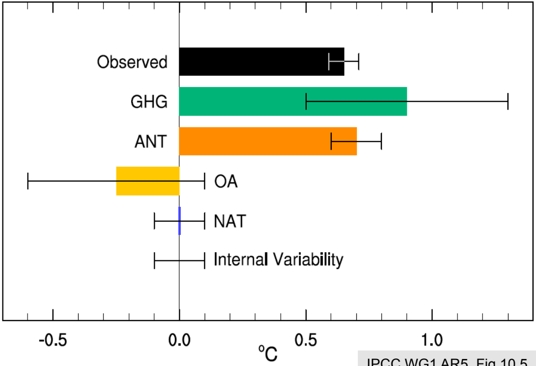
Source : Francis Zwiers

Calculating attributed change

Usual approach is to calculate trend in signal, multiply by scaling factor, and apply scaling factor uncertainty

Observed warming trend and 5-95% uncertainty range based on HadCRUT4 (black).

Attributed warming trends with assessed *likely* ranges (colours).



IPCC WG1 AR5, Fig 10.5

Source : Francis Zwiers

The basic Gaussian regression scheme

$$\hat{\beta} = \left(X^T \Sigma^{-1} X\right)^{-1} X^T \Sigma^{-1} Y$$

with under the Gaussian assumption with know Σ

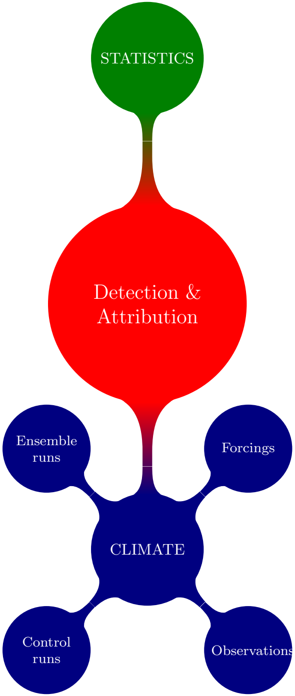
$$E(\hat{\beta}) = \beta \text{ and } Var(\hat{\beta}) = \left(X^T \Sigma^{-1} X\right)^{-1}$$

Practical questions

- $\beta = 0$ + CI ?
- $\beta = 1$ +CI ?

Problem done ? ... but

- What's about the dimension ?
- What's about the estimation of Σ^{-1} ?
- What's about the numerical models X ?


STATISTICS
Detection & Attribution
Ensemble runs
Forcings
CLIMATE
Control runs
Observations

What's about the dimension ?

Typical climate dataset (e.g. near-surface temperature)

- Spatial dimension : $5^o \times 5^o \approx 2600$ grid-points
- Temporal dimension : 50 - 100 ans (instrumental period)
- Dimension of Y$\approx 10^5$
- Internal variability is described by $\Sigma \approx 10^5 \times 10^5$

What's about the dimension ?

Typical climate dataset (e.g. near-surface temperature)

- Spatial dimension : $5^o \times 5^o \approx 2600$ grid-points
- Temporal dimension : 50 - 100 ans (instrumental period)
- Dimension of Y$\approx 10^5$
- Internal variability is described by $\Sigma \approx 10^5 \times 10^5$

Warming : Σ is not sparse because of teleconnections

- The estimation of Σ requires at least 10^5 realisations of ϵ, i.e. 10^7 yrs of control simulations (vs about $\approx 10^4$ yrs available).

What's about the dimension ?

Typical climate dataset (e.g. near-surface temperature)

- Spatial dimension : $5^o \times 5^o \approx 2600$ grid-points
- Temporal dimension : 50 - 100 ans (instrumental period)
- Dimension of Y$\approx 10^5$
- Internal variability is described by $\Sigma \approx 10^5 \times 10^5$

Warming : Σ is not sparse because of teleconnections

- The estimation of Σ requires at least 10^5 realisations of ϵ, i.e. 10^7 yrs of control simulations (vs about $\approx 10^4$ yrs available).

Two classical options

- Decrease the dimension of Y
- Find accurate estimator of Σ in large dimension

Source : Aurélien Ribes

Quick solutions

- Decadal means,
- Projection on principal components,
- Projection on spherical harmonics (e.g. truncation T4, $\approx$ spatial scales $>$ 5000 kms),
- Use of simple climate indices (globale mean, land-sea contrast, inter-hemispheric contrast, annual cycle, etc).

Source : Aurélien Ribes

– Recent studies (e.g., Jones et al, 2013) use
  - Gridded (5°×5°) monthly mean surface temperature anomalies (e.g., HadCRUT4, Morice et al, 2012)
  - Reduced to decadal means for 1901-1920, 1911-1920 … 2001-2010 (11 decades)
  - Often spatially reduced using a "T4" spherical harmonic decomposition ⇒ global array of 5°×5°decadal anomalies reduced to 25 coefficients
  - $\mathbf{Y}_{n\times 1}$ therefore has dimension n=11×25=275

Source : Francis Zwiers

What's about the covariates X ?

Signals $\mathbf{X}_i$, $i=1, \ldots, s$

- Number of signals s is small
  - s=1 → ALL
  - s=2 → ANT and NAT
  - s=3 → GHG, OANT and NAT
  - s=4 → …
- Can't separate signals that are "co-linear"
- Signals estimated from either
  - single model ensembles (size 3-10 in CMIP5) or
  - multi-model ensembles (~172 ALL runs available in CMIP5 from 49 models, ~67 NAT runs from 21 models , ~54 GHG runs from 20 models)
- Process as we do the observations
  - Transferred to observational grid, "masked", centered, averaged using same criteria, etc.

Source : Francis Zwiers

Still, we need to estimate the internal variability Σ

Is it a big deal ?

Still, we need to estimate the internal variability Σ

Is it a big deal ?

nature climate change

LETTERS

PUBLISHED ONLINE: 31 AUGUST 2014 | DOI: 10.1038/NCLIMATE2355

Contribution of natural decadal variability to global warming acceleration and hiatus

Masahiro Watanabe[1]*, Hideo Shiogama[2], Hiroaki Tatebe[3], Michiya Hayashi[1], Masayoshi Ishii[4] and Masahide Kimoto[1]

Reasons for the apparent pause in the rise of global-mean surface air temperature (SAT) after the turn of the century has been a mystery, undermining confidence in climate projections[1-3]. Recent climate model simulations indicate this warming hiatus originated from eastern equatorial Pacific cooling[4] associated with strengthening of trade winds[5]. Using a climate model that overrides tropical wind stress anomalies with observations for 1958–2012, we show that decadal-mean anomalies of global SAT referenced to the period 1961–1990 are changed by 0.11, 0.13 and −0.11 °C in the 1980s, 1990s and 2000s, respectively, without variation in human-induced radiative forcing. They account for about 47%, 38% and 27% of the respective temperature change. The dominant wind stress variability consistent with this warming/cooling represents the deceleration/acceleration of the Pacific trade winds, which can be robustly reproduced by atmospheric model simulations forced by observed sea surface temperature excluding anthropogenic warming components. Results indicate that inherent decadal climate variability contributes considerably to the observed global-mean SAT time series, but that its influence on decadal-mean SAT has gradually decreased relative to the rising anthropogenic warming signal.

The change of global-mean SAT during the first decade of the twenty-first century was less than 0.05 °C, indicating a considerably slower rate of warming than during the late twentieth century[3,6]. The causes of this global warming hiatus, which are still under debate, can be categorized into either internal or external processes of the climate system. The principal candidates for external drivers of the hiatus are the weakening of solar activity[7] and increase in stratospheric aerosols[8] plausibly associated with

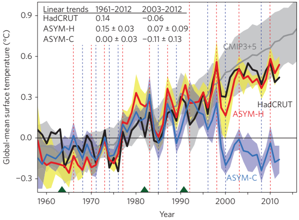


Figure 1 | Observed and simulated change in global-mean surface temperature. Annual-mean time series relative to 1961–1990 mean derived from observations (black), ASYM-H (red) and ASYM-C (blue) experiments. Shading represents ranges of 95% confidence. Linear trends for 1961–2012 and 2003–2012 are denoted at the top. Time series from the combined CMIP3 and CMIP5 models is also shown by the grey curve, with shading representing one standard deviation. Red and blue vertical dashed lines show the occurrence of El Niño and La Niña events, respectively. Three major volcanic eruptions (Agung, El Chichón and Pinatubo) are indicated by green triangles.

Still, we need to estimate the internal variability Σ

Climate models can provide

- $[\epsilon]$ = Control runs = a few simulations with constant (stationary) forcing that are used to estimate the so-called internal variability Σ
- Ensembles runs = a few GCM simulations with the same forcing but different initial conditions (give information on uncertainty associated with model error)

Notations : $[\epsilon] \sim N(0, \Sigma)$ *(also denoted* $\pi(\epsilon)$*) with dimension* $n \times r$ *and* $[\epsilon|y]$ *for conditional pdfs*

Still, we need to estimate the internal variability Σ

Climate models can provide

- $[\epsilon]$ = Control runs = a few simulations with constant (stationary) forcing that are used to estimate the so-called internal variability Σ
- Ensembles runs = a few GCM simulations with the same forcing but different initial conditions (give information on uncertainty associated with model error)

Notations : $[\epsilon] \sim N(0, \Sigma)$ *(also denoted* $\pi(\epsilon)$*) with dimension* $n \times r$ *and* $[\epsilon|y]$ *for conditional pdfs*

A fundamental statistical roadblock

The empirical estimator of the internal variability Σ

$$\hat{S} = \frac{1}{r}\epsilon\epsilon^T$$

is unbiased but has a very poor estimator if r small

Estimation of Σ

The idea of regularisation

$$\hat{\Sigma} = (1 - \alpha)\hat{S} + \alpha\Delta$$

with Δ is often chosen to be proportional to the identify matrix

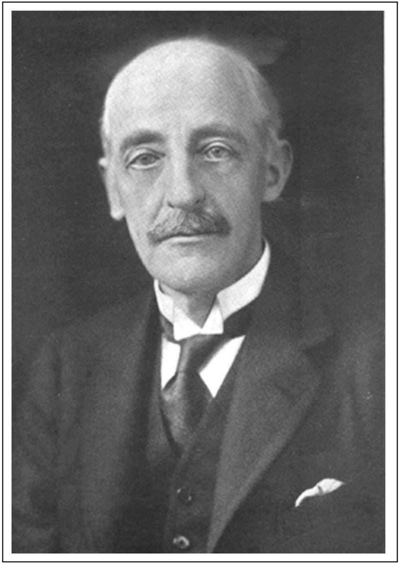
- Shrinkage estimator (LW04, Ledoit and Wolf, 2004)
- D&A see RPT12 Ribes A., S. Planton, L. Terray
- Link with James-Stein estimator
- Link with Bayesian a priori

Statistics and Earth sciences

"There is, today, always a risk that specialists in two subjects, using languages full of words that are unintelligible without study, will grow up not only, without knowledge of each other's work, but also will ignore the problems which require mutual assistance".

QUIZ

- *(A) Gilbert Walker*
- *(B) Ed Lorenz*
- *(C) Guillaume Maze*
- *(D) Rol Madden*



Pierre Simon Laplace (1749-1827)

“L’analyse des probabilités assigne la probabilité de ces causes, et elle indique les moyens d’accroitre de plus en plus cette probabilité.”
“Essai Philosophiques sur les probabilités” (1774)

Bayes' formula = calculating conditional probability

$$\boxed{[\theta|y] \propto [y|\theta] \times [\theta]}$$

REV. T. BAYES

1701(?)- 1761 "An essay towards solving a Problem in the Doctrine of Chances" (1764)

Recall of Gaussian basics

Let Z_1 and Z_2 a bivariate normal distribution with means μ_1 and μ_2 and a covariance matrix $\begin{bmatrix} \Sigma_{11} & \Sigma_{21} \\ \Sigma_{12} & \Sigma_{22} \end{bmatrix}$,

$$\begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} \sim N\left[\begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}, \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}\right]. \tag{1}$$

Conditioning

Then, the conditional distribution of Z_1 given Z_2 is described by

$$[Z_1|Z_2 = z_2] \sim N\left[\mu_1 + \Sigma_{12}\Sigma_{22}^{-1}(z_2 - \mu_2), \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}\right] \tag{2}$$

Estimating jointly β and Σ

$$\underbrace{p(\beta, \boldsymbol{\Sigma} \mid y, \boldsymbol{\varepsilon})}_{\text{a posteriori pdf of parameters } \beta \text{ and } \Sigma} \overset{\text{normalization factor}}{\propto} \underbrace{\mathcal{N}(y \mid x\beta, \boldsymbol{\Sigma}) \times \Pi_{i=1}^{r} \mathcal{N}(\varepsilon_i \mid \boldsymbol{\Sigma})}_{\text{update term from observations y and } \varepsilon \text{ (model likelihood)}} \times \underbrace{\pi(\beta) \times \pi(\boldsymbol{\Sigma})}_{\text{a priori pdf of parameters } \beta \text{ and } \Sigma}$$

Source : Alexis Hannart

Estimating jointly β and Σ

We use a uniform, improper prior for β:

$$p(\beta, \boldsymbol{\Sigma} \mid y, \boldsymbol{\varepsilon}) \propto \mathcal{N}(y \mid x\beta, \boldsymbol{\Sigma}) \times \Pi_{i=1}^{r} \mathcal{N}(\varepsilon_i \mid \boldsymbol{\Sigma}) \times \underbrace{\pi(\beta) \times \pi(\boldsymbol{\Sigma})}$$

⇒ a priori pdf of parameters β and Σ

⇒ $\pi(\beta) \propto 1$

Choosing an informative a priori pdf for $\Sigma \Leftrightarrow$ regularizing Σ

We now open a parenthesis to show that the choice of an informative prior for Σ corresponds to a linear regularization.

Let us return to the standard covariance model:

$$p(\boldsymbol{\Sigma} \mid \boldsymbol{\varepsilon}) \propto \Pi_{i=1}^{r} \mathcal{N}(\varepsilon_i \mid \boldsymbol{\Sigma}) \times \pi(\boldsymbol{\Sigma})$$

Choosing an informative a priori pdf for $\Sigma \Leftrightarrow$ regularizing Σ

Inverse Wishart Conjugate a priori pdf:

$$\begin{aligned}\pi(\boldsymbol{\Sigma}) &= \mathcal{W}^{-1}(\boldsymbol{\Sigma} \mid \nu, \boldsymbol{\Omega}) \\ &= 2^{-\frac{\nu n}{2}}\, \Gamma_n\left(\tfrac{\nu}{2}\right)^{-1} \, |\boldsymbol{\Omega}|^{\frac{\nu}{2}} \, |\boldsymbol{\Sigma}|^{-\frac{\nu+n+1}{2}} \exp\left\{-\tfrac{1}{2}\,\mathrm{Tr}(\boldsymbol{\Omega}\boldsymbol{\Sigma}^{-1})\right\} \\ &\propto |\boldsymbol{\Sigma}|^{-\frac{\alpha\, r}{2(1-\alpha)}-n-1} \exp\left\{-\tfrac{\alpha\, r}{2(1-\alpha)}\,\mathrm{Tr}(\boldsymbol{\Delta}\boldsymbol{\Sigma}^{-1})\right\}\end{aligned}$$

We reparameterize this conjugate prior in α and Δ

$$\begin{cases} (\alpha, \boldsymbol{\Delta}) = \left(\frac{\nu-n-1}{r+\nu-n-1}, \frac{\boldsymbol{\Omega}}{\nu-n-1}\right) \Leftrightarrow (\nu, \boldsymbol{\Omega}) = \left(\frac{\alpha\, r}{1-\alpha} + n + 1\,, \frac{\alpha\, r}{1-\alpha}\,\boldsymbol{\Delta}\right) \\ (\alpha, \boldsymbol{\Delta}) \in [0,1] \times \mathcal{S}^{+*} \end{cases}$$

Estimating jointly β and Σ

Choosing an informative a priori pdf for $\Sigma \Leftrightarrow$ regularizing Σ

a priori mean and variance under the Inverse Wishart pdf:

$$\mathbb{E}(\boldsymbol{\Sigma} \mid \alpha, \boldsymbol{\Delta}) = \boldsymbol{\Delta}$$

$$\mathrm{V}(\boldsymbol{\Sigma}_{ij} \mid \alpha, \boldsymbol{\Delta}) \simeq \tfrac{1-\alpha}{\alpha r}(\boldsymbol{\Delta}_{ij}^2 + \boldsymbol{\Delta}_{ii}\boldsymbol{\Delta}_{jj})$$

a posteriori mean under the Inverse Wishart pdf :

$$\mathbb{E}(\boldsymbol{\Sigma} \mid \boldsymbol{\varepsilon}, \alpha, \boldsymbol{\Delta}) = (1-\alpha)\widehat{\mathbf{S}} + \alpha\boldsymbol{\Delta} \qquad \left(\tfrac{1}{r}\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}' = \widehat{\mathbf{S}} \right)$$

Link with linear shrinkage towards identity (LW04):

choose $\Delta = \lambda \mathrm{I}$ and select optimal values for λ and α.

Choosing an a priori pdf for β and Σ

Returning to our model, we choose the Inverse Wishart Conjugate a priori pdf for Σ

$$p(\beta, \mathbf{\Sigma} \mid y, \boldsymbol{\varepsilon}) \propto \mathcal{N}(y \mid x\beta, \mathbf{\Sigma}) \times \Pi_{i=1}^{r} \mathcal{N}(\varepsilon_i \mid \mathbf{\Sigma}) \times \underbrace{\pi(\beta) \times \pi(\mathbf{\Sigma})}$$

a priori pdf of parameters β and Σ

$$\pi(\beta) \propto 1$$

$$\begin{cases} \pi(\mathbf{\Sigma} \mid \alpha, \mathbf{\Delta}) = \mathcal{W}^{-1}(\mathbf{\Sigma} \mid \alpha, \mathbf{\Delta}) \\ (\alpha, \mathbf{\Delta}) \in [0, 1] \times \mathcal{S}^{+*} \end{cases}$$

Deriving the marginal a posteriori pdf, mean and variance of β

After a few calculations to integrate out Σ, we obtain:

$$p(\beta \mid y, \boldsymbol{\varepsilon}) = \mathcal{T}(\beta \mid \hat{\beta}, \widehat{\boldsymbol{\Omega}}, \nu + r + 1 - p) \simeq \mathcal{N}(\beta \mid \hat{\beta}, \widehat{\boldsymbol{\Omega}})$$

And the following estimators of β, its variance, and Σ:

$$\begin{cases} \hat{\beta} = (x'\widehat{\boldsymbol{\Sigma}}^{-1}x)^{-1}(x'\widehat{\boldsymbol{\Sigma}}^{-1}y) \\ \widehat{\boldsymbol{\Omega}} = \dfrac{1+\frac{1-\alpha}{r}\, y'\,(\widehat{\boldsymbol{\Sigma}}^{-1} - \widehat{\boldsymbol{\Sigma}}^{-1}x(x'\widehat{\boldsymbol{\Sigma}}^{-1}x)^{-1}x'\widehat{\boldsymbol{\Sigma}}^{-1})\, y}{1+\frac{1-\alpha}{r}\,(n-p)} \,.\, (x'\widehat{\boldsymbol{\Sigma}}^{-1}x)^{-1} \\ \widehat{\boldsymbol{\Sigma}} = \alpha\boldsymbol{\Delta} + (1-\alpha)\widehat{\mathbf{S}} \end{cases}$$

Estimating jointly β and Σ

Deriving the marginal a posteriori pdf, mean and variance of β

$$\begin{cases} \hat{\beta} = (x'\widehat{\boldsymbol{\Sigma}}^{-1}x)^{-1}(x'\widehat{\boldsymbol{\Sigma}}^{-1}y) \\ \widehat{\boldsymbol{\Omega}} = \frac{1+\frac{1-\alpha}{r}\, y'\,(\widehat{\boldsymbol{\Sigma}}^{-1}-\widehat{\boldsymbol{\Sigma}}^{-1}x(x'\widehat{\boldsymbol{\Sigma}}^{-1}x)^{-1}x'\widehat{\boldsymbol{\Sigma}}^{-1})\,y}{1+\frac{1-\alpha}{r}\,(n-p)} \,.\, (x'\widehat{\boldsymbol{\Sigma}}^{-1}x)^{-1} \\ \widehat{\boldsymbol{\Sigma}} = \alpha\boldsymbol{\Delta} + (1-\alpha)\widehat{\mathbf{S}} \end{cases}$$

⇒ The estimator of β is the same as the one proposed by RPT12. This gives further theoretical grounding to this estimator.

⇒ However, the estimator of its variance differs, as it includes a scaling factor.

RPT12 : Ribes A., S. Planton, L. Terray

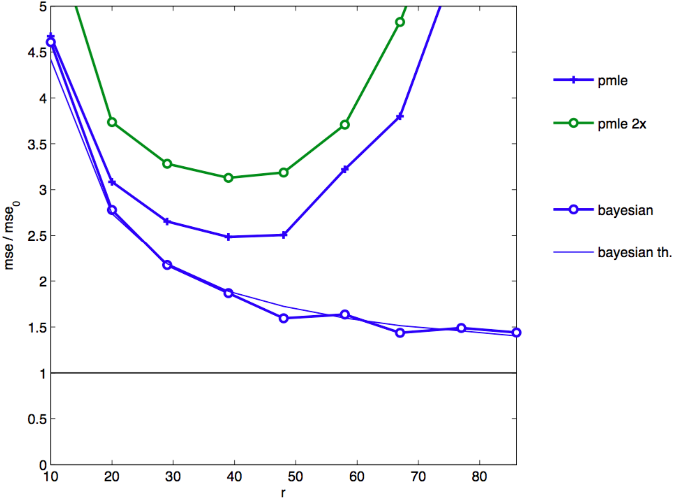
Simulation-based performance comparison

- Simulations:
  - n = 100, p = 3, r = 10, 20, …, 100.
  - $\beta = (1,\ldots,1)$.
  - Σ, x randomly generated from Inverse Wishart pdf and Gaussian pdf.
  - y, ε randomly generated from model assumptions

- Performance metrics:
  - empirical mse $= \frac{1}{N}\sum_{i=1}^{N}(\hat{\beta}_i - \beta)'(\hat{\beta}_i - \beta)$
  - theoretical mse $= \frac{1}{N}\sum_{i=1}^{N}\mathbb{E}\left((\hat{\beta}_i - \beta)'(\hat{\beta}_i - \beta)\right) = \frac{1}{N}\sum_{i=1}^{N}\mathrm{Tr}(\widehat{\boldsymbol{\Omega}}_i)$
  - normalization to empirical mse with known Σ.

Estimating jointly β and Σ

When α is known (here =0.6), the Bayesian estimator outperforms both mle. The estimate of its variance is unbiased.



Estimating jointly β and Σ

However, in practice, α is usually not known. The LW04 approach is able to yield an estimate only when $\Delta = \lambda I$.

- Ledoit and Wolf 2004, JMVA:
  - optimal value of α for a target Δ proportional to the identity
  - a few extensions in very specific cases of Δ (later on)
  - no general expression available for Δ unspecified

Estimating jointly β and Σ

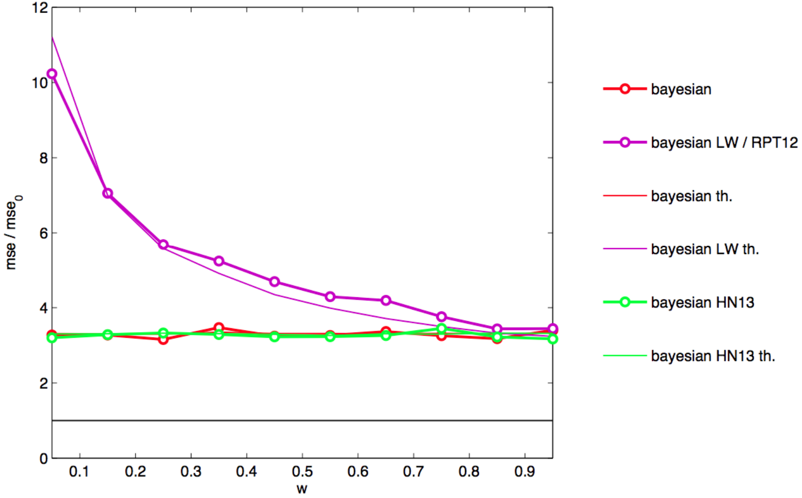
For a general Δ, we use instead the following estimate for α

- Hannart and Naveau 2013, submitted to JMVA:
  - optimal value of α for any target Δ:

$$\alpha^* = \mathrm{argmax}_{\alpha\in[0,1]} \quad (\tfrac{\alpha r}{1-\alpha} + n + 1)\log|\tfrac{\alpha}{1-\alpha}\mathbf{\Delta}| - (\tfrac{r}{1-\alpha} + n + 1)\log|\widehat{\mathbf{S}} + \tfrac{\alpha}{1-\alpha}\mathbf{\Delta}|$$
$$+ \quad 2\log\left(\Gamma_n\{\tfrac{1}{2}(\tfrac{r}{1-\alpha} + n + 1)\} / \Gamma_n\{\tfrac{1}{2}(\tfrac{\alpha r}{1-\alpha} + n + 1)\}\right)$$

Estimating jointly β and Σ

The obtained Bayesian estimator with estimated α now achieves the same performance as the Bayesian estimator with known α.



What's about the GCM ? (Source : IPCC AR5 WG1)

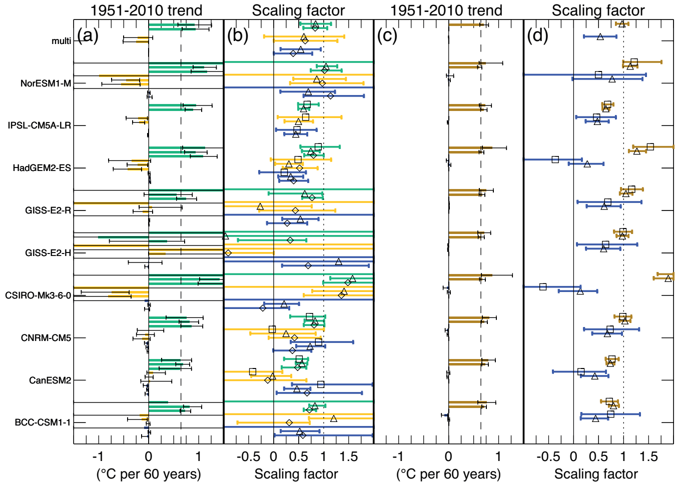


Figure 10.4 | (a) Estimated contributions of greenhouse gas (GHG, green), other anthropogenic (yellow) and natural (blue) forcing components to observed global mean surface temperature (GMST) changes over the 1951–2010 period. (b) Corresponding scaling factors by which simulated responses to GHG (green), other anthropogenic (yellow) and natural forcings (blue) must be multiplied to obtain the best fit to Hadley Centre/Climatic Research Unit gridded surface temperature data set 4 (HadCRUT4; Morice et al., 2012) observations based on multiple regressions using response patterns from nine climate models individually and multi-model averages (multi). Results are shown based on an analysis over the 1901–2010 period (squares, Ribes and Terray, 2013), an analysis over the 1861–2010 period (triangles, Gillett et al., 2013) and an analysis over the 1951–2010 period (diamonds, Jones et al., 2013). (c, d) As for (a) and (b) but based on multiple regressions estimating the contributions of total anthropogenic forcings (brown) and natural forcings (blue) based on an analysis over 1901–2010 period (squares, Ribes and Terray, 2013) and an analysis over the 1861–2010 period (triangles, Gillett et al., 2013). Coloured bars show best estimates of the attributable trends (a and c) and 5 to 95% confidence ranges of scaling factors (b and d). Vertical dashed lines in (a) and (c) show the best estimate HadCRUT4 observed trend over the period concerned. Vertical dotted lines in (b) and d) denote a scaling factor of unity.

Internal variability within the GCM X

A new source of uncertainty

The matrix of actual regressors $\mathbf{x}^*$ of size $n \times p$ is not known with certainty and the observed matrix $\mathbf{x}$ is assumed to be a noised version of it

$$\mathbf{x} = \mathbf{x}^* + \nu$$

where $[\nu_i] \sim N(0, \Omega_i)$

A new source of uncertainty

The matrix of actual regressors $\mathbf{x}^*$ of size $n \times p$ is not known with certainty and the observed matrix $\mathbf{x}$ is assumed to be a noised version of it

$$\mathbf{x} = \mathbf{x}^* + \nu$$

where $[\nu_i] \sim N(0, \Omega_i)$

A difficult problem to solve

Even with only one regressors $p = 1$, this is a non-parametric problem with n unknowns and an unknown matrix Ω of size $n \times n$

Error-In-Variable model (EIV)

A new system with four unknowns $\beta, \mathbf{x}^*, \Omega$ and Σ

$$\left\{\begin{array}{ll} \mathbf{y} & = \mathbf{x}^*\beta + \epsilon, \text{ with } \epsilon \sim \mathcal{N}_n(\mathbf{0}, \Sigma), \\ \mathbf{x} & = \mathbf{x}^* + \nu, \text{ with } \nu \sim \mathcal{N}_n(\mathbf{0}, \Omega), \end{array}\right.$$

Error-In-Variable model (EIV)

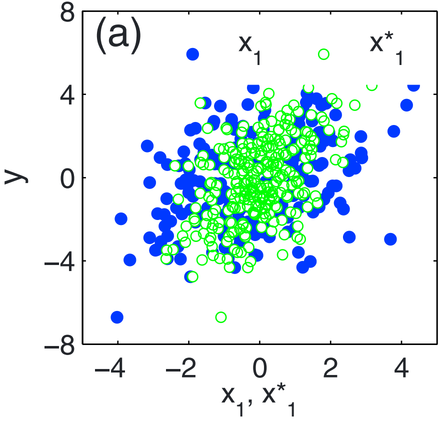
A new system with four unknowns $\beta, \mathbf{x}^*, \Omega$ and Σ

$$\begin{cases} \mathbf{y} & = \mathbf{x}^*\beta + \epsilon, \text{ with } \epsilon \sim \mathcal{N}_n(\mathbf{0}, \Sigma), \\ \mathbf{x} & = \mathbf{x}^* + \nu, \text{ with } \nu \sim \mathcal{N}_n(\mathbf{0}, \Omega), \end{cases}$$

A short bibliography

- No known solution for the general case
- Univariate EIV Adcock [1878] & Gillard [2010]
- $\nu = \mathbf{0}$ in the D&A, see Allen & Tett (1999)
- $\Omega_i = \Sigma / n_r$ Allen & Stott (2003)
- $\Omega_i = \Delta + \Sigma / n_r$ Huntingford et al. (2006)
- Covariances estimation (Ribes, A., S. Planton, and L. Terray (2012)), classically a two-step plugging.

Error-In-Variable model (EIV)



EIV with known covariances (Source : Hannart, Ribes, Naveau, GRL, 2014))

EIV system

$$\begin{cases} \mathbf{y} & = \mathbf{x}^* \beta + \epsilon, \text{ with } \epsilon \sim \mathcal{N}_n(\mathbf{0}, \Sigma), \\ \mathbf{x} & = \mathbf{x}^* + \nu, \text{ with } \nu \sim \mathcal{N}_n(\mathbf{0}, \Omega), \end{cases}$$

Likelihood function

$$\ell(\beta, \mathbf{x}^* \mid y, \mathbf{x}) = -\frac{1}{2}(y - \mathbf{x}^*\beta)'\mathbf{\Sigma}^{-1}(y - \mathbf{x}^*\beta) - \frac{1}{2}\sum_{i=1}^{p}(x_i - x_i^*)'\mathbf{\Omega}_i^{-1}((x_i - x_i^*).$$

MLE equations

$$\beta = (\mathbf{x}^{*\prime}\mathbf{\Sigma}^{-1}\mathbf{x}^*)^{-1}(\mathbf{x}^{*\prime}\mathbf{\Sigma}^{-1}y)$$

$$x_i^* = \left(\mathbf{\Omega}_i^{-1} + \beta_i^2\mathbf{\Sigma}^{-1}\right)^{-1}\left(\beta_i\mathbf{\Sigma}^{-1}\overline{y}_i + \mathbf{\Omega}_i^{-1}x_i\right) \quad \text{for } i = 1, ..., p$$

EIV with known covariances (Source : Hannart, Ribes, Naveau, GRL, 2014))

EIV system

$$\begin{cases} \mathbf{y} &= \mathbf{x}^* \beta + \epsilon, \text{ with } \epsilon \sim \mathcal{N}_n(\mathbf{0}, \Sigma), \\ \mathbf{x} &= \mathbf{x}^* + \nu, \text{ with } \nu \sim \mathcal{N}_n(\mathbf{0}, \Omega), \end{cases}$$

A Gibbs type algorithm

$$\begin{cases} \bullet \text{ initialization: } \mathbf{x}^{*(0)} = \mathbf{x} \textit{ and } \beta^{(0)} = (\mathbf{x}'\mathbf{\Sigma}^{-1}\mathbf{x})^{-1}(\mathbf{x}'\mathbf{\Sigma}^{-1}y). \\ \bullet \text{ iterationstep1: } x_i^{*(t+1)} = (\mathbf{\Omega}_i^{-1} + \beta_i^{(t)2}\mathbf{\Sigma}^{-1})^{-1}(\beta_i^{(t)}\mathbf{\Sigma}^{-1}\overline{y}_i^{(t)} + \mathbf{\Omega}_i^{-1}x_i) foreachi. \\ \bullet \text{ iterationstep2: } \beta^{(t+1)} = (\mathbf{x}^{*(t+1)\prime}\mathbf{\Sigma}^{-1}\mathbf{x}^{*(t+1)})^{-1}(\mathbf{x}^{*(t+1)\prime}\mathbf{\Sigma}^{-1}y). \\ \bullet \text{ stopping: repeat iterations until } \|\beta^{(t+1)} - \beta^{(t)}\| / \|\beta^{(t)}\| < \delta_0. \end{cases}$$

Confidence intervals for β_i

Derived from the profile likelihood

$$\ell_i(\beta_i \mid y, \mathbf{x}) = \max_{(\beta_{-i}, \mathbf{x}^*)} \ell(\beta, \mathbf{x}^* \mid y, \mathbf{x})$$

EIV with known covariances (Source : Hannart, Ribes, Naveau, GRL, 2014))

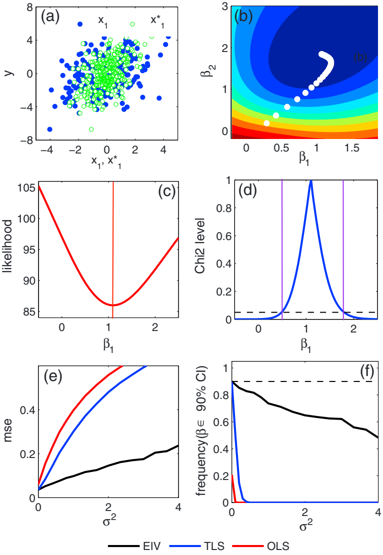


Figure 1. (a–d) Illustration of the inference procedure for a simulation $n = 275$, $p = 2$, and $\sigma = \sigma_0$ and (e and f) performance results. Data scatterplot (x_1, y) (blue dots) and (x_1^*, y) (green circles) shown in Figure 1a. Contour plot of the negative profile loglikelihood $-\ell_i(\beta)$ and trajectory of $\beta^{(t)}$ showing convergence to the minimum shown in Figure 1b. Plot of the negative profile loglikelihood $-\ell_i(\beta_1)$ shown in Figure 1c. Plot of the χ^2 probability level and confidence interval shown in Figure 1d. Average mean squared error of the estimator obtained with our procedure (EIV, black line), TLS (blue line), and OLS (red line) shown in Figure 1f. Frequency of the actual value of β falling within the 90% confidence interval for our procedure, TLS, and OLS shown in Figure 1f.

EIV statistical challenges

Classical system

$$\begin{cases} \mathbf{y} & = \mathbf{x}^*\beta + \epsilon, \text{ with } \epsilon \sim \mathcal{N}_n(\mathbf{0}, \Sigma), \\ \mathbf{x} & = \mathbf{x}^* + \nu, \text{ with } \nu \sim \mathcal{N}_n(\mathbf{0}, \Omega), \end{cases}$$

Inference difficulty

Estimating jointly $\beta, \mathbf{x}^*, \Omega$ and Σ, see our previous section of the Whishart's prior on Σ

EIV statistical challenges

Classical system

$$\begin{cases} \mathbf{y} & = \mathbf{x}^* \beta + \epsilon, \text{ with } \epsilon \sim \mathcal{N}_n(\mathbf{0}, \Sigma), \\ \mathbf{x} & = \mathbf{x}^* + \nu, \text{ with } \nu \sim \mathcal{N}_n(\mathbf{0}, \Omega), \end{cases}$$

Inference difficulty

Estimating jointly $\beta, \mathbf{x}^*, \Omega$ and Σ, see our previous section of the Whishart's prior on Σ

Possible new model definition (ongoing research)

$$\text{M} \begin{cases} \mathbf{y} & = \mathbf{y}^* + \epsilon, \text{ with } \epsilon \sim \mathcal{N}_n(\mathbf{0}, \Sigma), \\ \mathbf{x} & = \mathbf{x}^* + \nu, \text{ with } \nu \sim \mathcal{N}_n(\mathbf{0}, \Omega), \end{cases} \ \& \ \tilde{M} \begin{cases} \mathbf{y} & = \tilde{\mathbf{y}}^* + \tilde{\epsilon}, \text{ with } \epsilon \sim \mathcal{N}_n(\mathbf{0}, \tilde{\Sigma}), \\ \tilde{\mathbf{x}} & = \tilde{\mathbf{x}}^* + \tilde{\nu}, \text{ with } \nu \sim \mathcal{N}_n(\mathbf{0}, \tilde{\Omega}), \end{cases}$$

with

$$[\mathbf{x}^* | \mu, \Delta] \sim \mathcal{N}_n(\mu, \Delta) \text{ and } [\mathbf{y}^* | \mu, \Delta] \sim \mathcal{N}_n(\mu, \Delta).$$

EIV statistical challenges

Classical system

$$\begin{cases} \mathbf{y} & = \mathbf{x}^*\beta + \epsilon, \text{ with } \epsilon \sim \mathcal{N}_n(\mathbf{0}, \Sigma), \\ \mathbf{x} & = \mathbf{x}^* + \nu, \text{ with } \nu \sim \mathcal{N}_n(\mathbf{0}, \Omega), \end{cases}$$

Inference difficulty

Estimating jointly $\beta, \mathbf{x}^*, \Omega$ and Σ, see our previous section of the Whishart's prior on Σ

Possible new model definition (ongoing research)

$$M \begin{cases} \mathbf{y} & = \mathbf{y}^* + \epsilon, \text{ with } \epsilon \sim \mathcal{N}_n(\mathbf{0}, \Sigma), \\ \mathbf{x} & = \mathbf{x}^* + \nu, \text{ with } \nu \sim \mathcal{N}_n(\mathbf{0}, \Omega), \end{cases} \quad \& \ \tilde{M} \begin{cases} \mathbf{y} & = \tilde{\mathbf{y}}^* + \tilde{\epsilon}, \text{ with } \epsilon \sim \mathcal{N}_n(\mathbf{0}, \tilde{\Sigma}), \\ \tilde{\mathbf{x}} & = \tilde{\mathbf{x}}^* + \tilde{\nu}, \text{ with } \nu \sim \mathcal{N}_n(\mathbf{0}, \tilde{\Omega}), \end{cases}$$

with

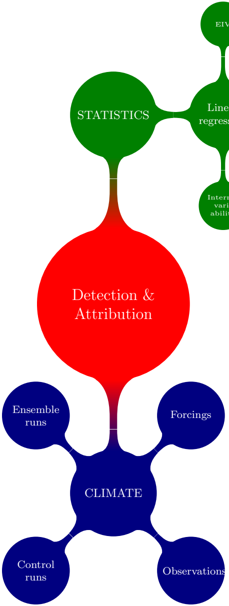
$$[\mathbf{x}^* | \mu, \Delta] \sim \mathcal{N}_n(\mu, \Delta) \text{ and } [\mathbf{y}^* | \mu, \Delta] \sim \mathcal{N}_n(\mu, \Delta).$$

Goal : computing the Bayes factor

The posteriors odds ratio

$$B_{M,\tilde{M}} = \frac{[M|\mathbf{y}]}{[\tilde{M}\ \mathbf{y}]} = ?$$

compares the models M and $\tilde{M}$


EIV
STATISTICS
Linear regression
Internal vari- ability
Detection & Attribution
Ensemble runs
Forcings
CLIMATE
Control runs
Observations

Back to D&A

Attribution

Evaluating the relative contributions of multiple causal factors [3] to a change or event with an assignment of statistical confidence.

---

3. casual factors usually refer to external influences, which may be anthropogenic (GHGs, aerosols, ozone precursors, land use) and/or natural (volcanic eruptions, solar cycle modulations

Questions for D&A

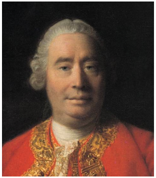
- Is it possible to define « causality » more precisely ?
- Is it possible to quantify « causal evidence » more rigorously ?
- Are the causal claims regarding the anthropogenic influence on climate justified ?
- Can we formulate a unified « causal evidencing framework » for climate science ?

Coming slides : Hannart, A., Pearl J. Otto F., P. Naveau and M. Ghil. (submitted). Counterfactual causality theory for the attribution of weather and climate-related events

The cornerstone of causality: counterfactual definition

- D. Hume, *An Enquiry Concerning Human Understanding*,1748

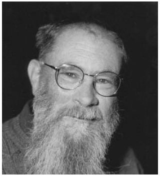
  « We may define a cause to be an object followed by another, where, if the first object had not been, the second never had existed. »



D. Hume, 18th century

- D. K. Lewis, *Counterfactuals*, 1973

  « We think of a cause as something that makes a difference, and the difference it makes must be a difference from what would have happened without it. Had it been absent, its effects would have been absent as well. »



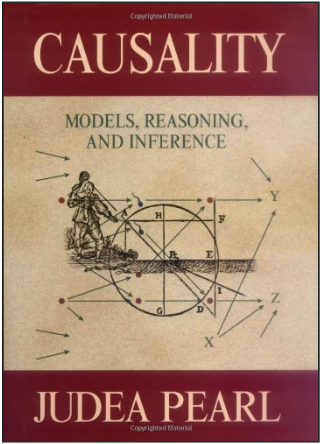
D. Lewis, 20th century

Consolidation of a standard causality theory (1980-1990)

- Common theoretical corpus on causality
  - what does «X causes Y» mean ?
  - how does one evidence a causality link from data ?
  - philosophy, artificial intelligence, statistics.
  - statistics alone not enough - more concepts needed.

- J. Pearl (2000), *Causality: models, reasoning and inference*, Cambridge University Press.

- Turing Award 2004.



- Provides clear semantics and sound logic for causal reasoning.

Conditional probability at work

- Let X, Y, Z be random variables (e.g. binary).
  - X: barometer
  - Y: rain
  - Z: road wet
  - W: low pressure system

$$\Rightarrow \begin{cases} P(Z \mid X, Y) = P(Z \mid Y) \\ P(Z \mid Y, W) = P(Z \mid Y) \\ P(Y \mid X, W) = P(Y \mid W) \end{cases}$$

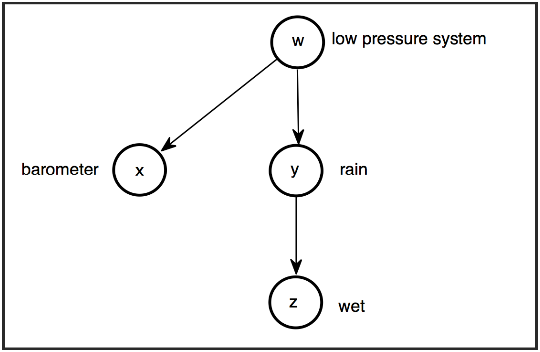
Dependence hierachy

- Let X, Y, Z be random variables (e.g. binary).
  - X: barometer
  - Y: rain
  - Z: road wet
  - W: low pressure system

$$P(X,Y,Z,W) = P(W)\,.\,P(X \mid W)\,.\,P(Y \mid W)\,.\,P(Z \mid Y)$$

Oriented graphs

— visual representation of the conditional independence structure of a joint distribution



$$P(X,Y,Z,W) = P(W) \,.\, P(X \mid W) \,.\, P(Y \mid W) \,.\, P(Z \mid Y)$$

Interventional probability

- Limitation of oriented graphs
  - identifiability: several causal graphs are compatible with the same pdf (and hence with the same observations).

$$P(X,Y) = P(X).P(Y \mid X) = P(Y).P(X \mid Y)$$

$$X \to Y \qquad\qquad Y \to X$$

  - Need for disambiguation.

<u>**experimentation**</u>

Interventional probability

- New notion:
  - intervention $do(X=x)$
  - interventional probability $P(Y \mid do(X=x)) = P(Y_x)$

the probability of rain **forcing** the barometer to decrease,
in an experimental context in which the barometer is manipulated

$$P(Y \mid do(X=x)) \neq P(Y \mid X=x)$$

the probability of rain **knowing** that the barometer is decreasing,
in a non-experimental context in which the barometer evolution is left unconstrained

Interventional probability

- Property:
  - Exogeneity: X exogenous if X has no parents
  - in this case:

$$P(Y \mid do(X = x)) = P(Y \mid X = x)$$

Fundamental difference : necessary and sufficient causation

- Definitions:
  - "*X is a necessary cause of Y*" means that X is required for Y to occur but that other factors might be required as well.
  - "*X is a sufficient cause of Y*" means that X always triggers Y but that Y may also occur for other reasons without requiring X.
- Examples:
  - clouds are a necessary cause of rain but not a sufficient one.
  - rain is a sufficient cause for the road being wet, but not a necessary one.

Fundamental difference : necessary and sufficient causation

- Definitions:
  - **<u>Probability of necessary causality = PN</u>** = the probability that the event Y would not have occurred in the absence of the event X given that both events Y and X did in fact occur.
  - **<u>Probability of sufficient causation = PS</u>** = the probability that Y would have occurred in the presence of X, given that Y and X did not occur.
- Formalization:

$$\begin{cases} \mathrm{PN} =_{\text{def}} P(Y_0 = 0 \mid Y = 1, X = 1) \\ \mathrm{PS} =_{\text{def}} P(Y_1 = 1 \mid Y = 0, X = 0) \\ \mathrm{PNS} =_{\text{def}} P(Y_0 = 0, Y_1 = 1) \end{cases}$$

Necessary and sufficient causation

- How to calculate PN, PS and PNS ?
  - difficult in general.
  - closed formula under the assumption of <u>monotonicity</u>:

$$
\begin{cases}
\mathrm{PN} = 1 - \frac{p_0}{p_1} + \frac{p_0 - P(Y_0=1)}{P(X=1, Y=1)} \\
\mathrm{PS} = 1 - \frac{1-p_1}{1-p_0} - \frac{p_1 - P(Y_1=1)}{P(X=0, Y=0)} \\
\mathrm{PNS} = P(Y_1 = 1) - P(Y_0 = 1)
\end{cases}
$$

where:
p1 = P(Y=1 I X = 1) : factual probability of the event
p0 = P(Y=1 I X = 0) : counterfactual probability of the event

Necessary and sufficient causation

- How to calculate PN, PS and PNS ?
  - difficult in general
  - closed formula under assumption of monotonicity
  - simplifies further under monotonicity and exogeneity:

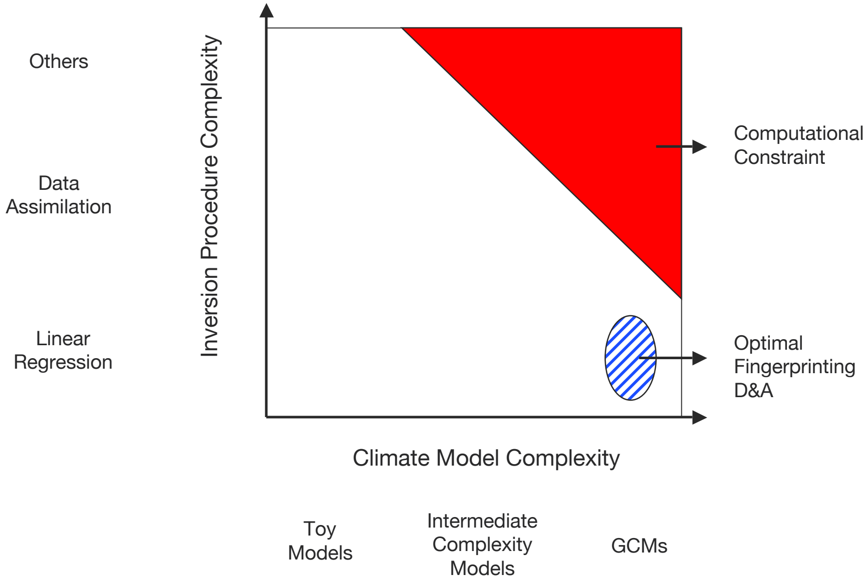
$$\mathrm{PN} = 1 - \frac{p_0}{p_1}, \quad \mathrm{PS} = 1 - \frac{1 - p_1}{1 - p_0}, \quad \mathrm{PNS} = p_1 - p_0$$

FAR, « excess risk ratio »

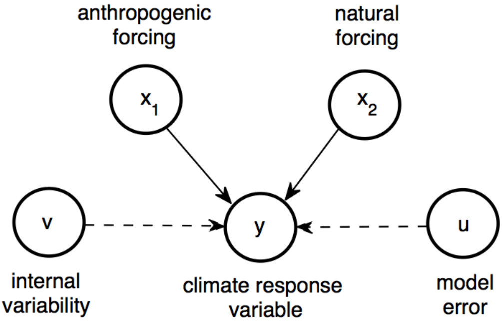
Recall : The FAR = the relative ratio of two probabilities, p_0 the probability of exceeding a threshold in a “world that might have been (no antropogenic forcings)” and p_1 the probability of exceeding the same threshold in a “world that it is”

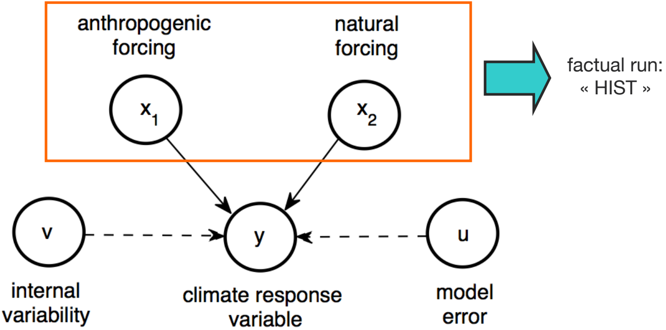
$$FAR = \frac{p_1 - p_0}{p_1}$$

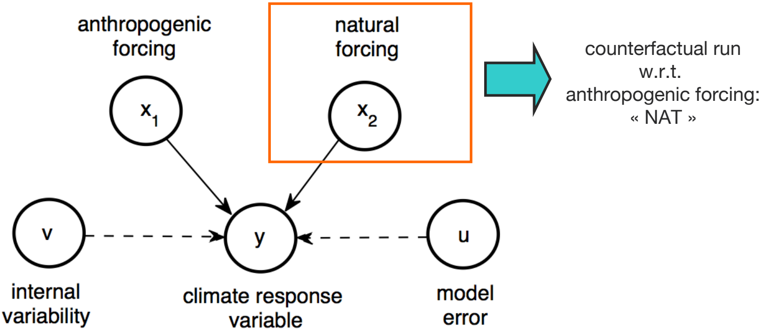
Big data : statistical versus numerical models

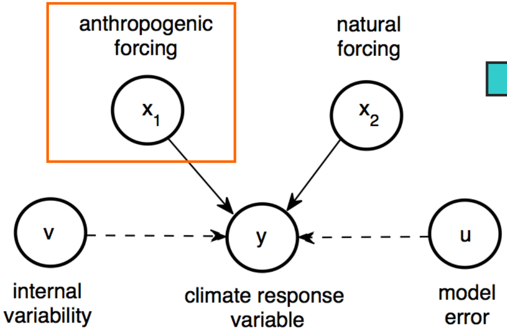


Back to climate sciences




anthropogenic forcing
natural forcing
x_1
x_2
factual run:
« HIST »
v
y
u
internal variability
climate response variable
model error

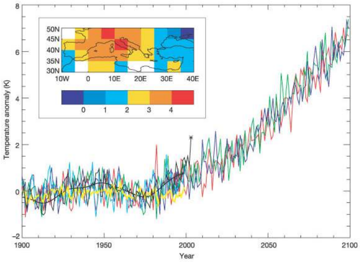

anthropogenic forcing
natural forcing
x1
x2
counterfactual run w.r.t. anthropogenic forcing: « NAT »
v
y
u
internal variability
climate response variable
model error



counterfactual run w.r.t. natural forcing: « ANT »

The 2003 European heatwave Stott P. A., Stone D. A., Allen M. R. (2004). Human contribution to the European heatwave of 2003. Nature,

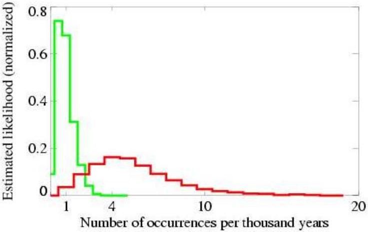
Methodology (1)

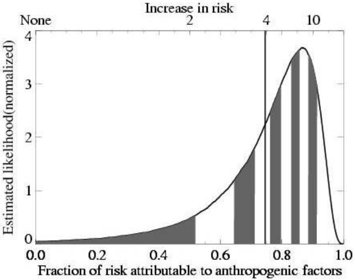


- Analyse JJA mean temperatures over a previously defined region that includes Central Europe
- Select an extreme temperature threshold just above the previous warmest year
- Determine mean temperature in “world that is” and compare to mean temperature in “world that might have been”
- By analysing the year to year variability around the mean climate in the two worlds calculate the probabilities P1, P0 of exceeding the threshold in the two worlds

The 2003 European heatwave Stott P. A., Stone D. A., Allen M. R. (2004). Human contribution to the European heatwave of 2003. Nature,

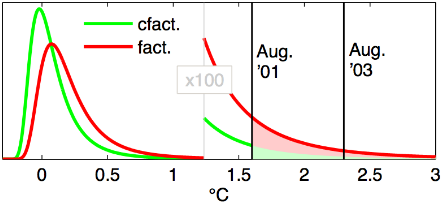
“Using a threshold for mean summer temperature that was exceeded in 2003, but in no other year since the start of the instrumental record in 1851, we estimate it is very likely (confidence level >90%) that human influence has at least doubled the risk of a heatwave exceeding this threshold magnitude”





Revisiting the 2003 European heatwave with counterfactual theory

EVT extrapolation (GEV) based on HIST and NAT ensembles (Hadley center model)



p0 = 0.0008 (1/1250), p1 = 0.008 (1/125)

$p0 = 0.0008$ (1/1250), $p1 = 0.008$ (1/125)

PN = 0.9, PS = 0.0072, PNS = 0.0072

« CO2 emissions are very likely to be a necessary cause, but are virtually certainly not a sufficient cause, of the 2003 heatwave. »

This highlights a distinctive feature of unusual events: several necessary causes may often be evidenced but rarely a sufficient one

« It is very likely (>90%) that CO_2 emissions have increased the frequency of occurrence of 2003-like heatwaves by a factor at least two »

$\neq$

« CO_2 emissions are very likely to be a necessary cause of the 2003 heatwave. »

Event attribution - summary

- *« Have CO2 emissions caused the 2003 European heatwave? »*

- The answer is greatly affected by:
  - how one defines the event « 2003 European heatwave »,
  - what is the temporal focus of the question,
  - whether causality is understood in a necessary or sufficient sense.

Precise causal answers about climate events critically require precise causal questions.

- Which matters for event attribution: PN, PS or PNS ?

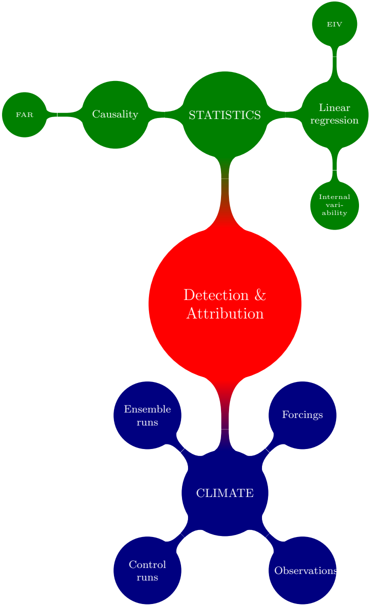
- The ex post perspective (judge) :
  - «who is to blame for the weather event that occurred ?»
  - insurance, compensation, loss and damage mechanisms (e.g. Warsaw 2013)
  - PN matters, not PS.

- The ex ante perspective (policy maker)
  - «what should be done today w.r.t. events that may occur in the future?»
  - PS matters for assessing the cost of inaction, PN for assessing the benefit of action.

- The dissemination perspective (media, IPCC)
  - PNS is a trade off between PN and PS.
  - good candidate for a single metric as it avoids explaining the distinction.

⇒ **PN, PS and PNS all matter**

Summary



EIV
FAR
Causality
STATISTICS
Linear regression
Internal vari- ability
Detection & Attribution
Ensemble runs
Forcings
CLIMATE
Control runs
Observations

Statistical challenges

- Making links with other communities (machine learning, data mining, ...)
- Reframing FAR D&A questions and definitions by injecting error models
- Investigating further regression models within the counterfactual theory
- Finding ways to estimate non-sparse and big covariance matrices
- Moving away from the Gaussian framework for extremes
- Uncertainty of FAR as the ratio of two small probabilities
- Adding more physics within the statistical model (data assimilation)
- Taking advantage of fast algorithms
- Add a Bayesian flavor to clarify assumptions
- Improve climate models and their use (design experiments)

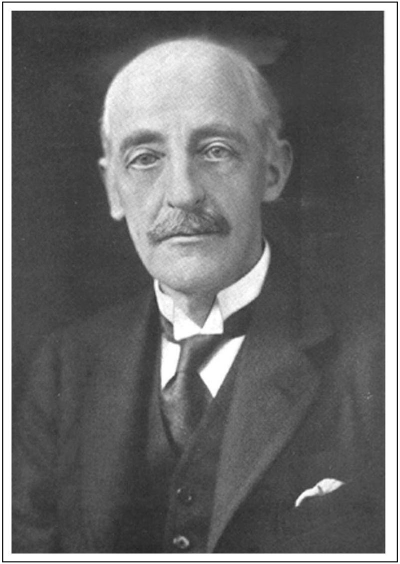
A very short bibliography

- Allen, M.R., and S.F.B. Tett, 1999 : Checking for model consistency in optimal fingerprinting. Clim. Dyn., 15, 419–434. 3.
- Allen, M.R., and P.A. Stott, 2003 : Estimating signal amplitudes in optimal fingerprinting, Part I : Theory. Clim. Dyn., 21, 477–491.
- Hannart, A., A. Ribes, and P. Naveau, 2014 : Optimal fingerprinting under multiple sources of uncertainty, Geophys. Res. Lett., 41, 1261–1268.
- Hannart, A. and P. Naveau, 2014. Estimating high dimensional covariance matrices : A new look at the Gaussian conjugate framework J. Multi. Analysis.
- Hannart, A., Pearl J. Otto F., P. Naveau and M. Ghil. (submitted). Counterfactual causality theory for the attribution of weather and climate-related events
- Huntingford,C.,P.A.Stott,M.R.Allen,andF.H.Lambert,2006 :Incorporating model uncertainty into attribution of observed temperature change. Geophys. Res. Lett., 33.
- Ledoit O,Wolf M(2004),A well-conditioned estimator for large dimensional covariance matrices. J. Multivar. Analysis. ,88 :365-411
- J. Pearl (2000), Causality : models, reasoning and inference, Cambridge University Press.
- Ribes A, Planton S, Terray L,2013a : Application of regularized optimal fingerprinting to attribution. Clim Dyn, doi :10.1007/s00382-013-1735-7
- Hegerl, G. C., et al., 2010 : Good practice guidance paper on detection and attribution related to anthropogenic climate change. IPCC WG1.
- IPCC, 2013, The physical science basis. WG1
- Zhang,X.,H. Wan,F.W. Zwiers, G.C. Hegerl, and S.-K.Min,2013 :Attributing intensification of precipitation extremes to human influence, Geophys. Res. Lett., 40, 5252–5257, doi :10.1002/grl.51010.

"There is, today, always a risk that specialists in two subjects, using languages full of words that are unintelligible without study, will grow up not only, without knowledge of each other's work, but also will ignore the problems which require mutual assistance".

QUIZ

- *(A) Gilbert Walker*



Necessary and sufficient causation

- The judge perspective:
  - defendant A shot a gun at random in a seemingly desert place.
  - B stood one kilometer away and was unluckily hit right in between the eyes.
  - PN ~ 1, PS ~ 0.
  - but A is an obvious culprit for the death of B from a legal perspective.
  - only PN matters here, PS does not.

- The policy-maker perspective:
  - what is the best policy to achieve a given objective ? (say, reducing accidental gunshot mortality)
    - prohibiting guns sales => PN = .., PS ~ 1
    - restricting guns sales => PN = .., PS = …
    - better informing gun owners on safety => PN = .., PS = …
  - both PN and PS matter to assess efficiency.